Estimating Solar Radiation Received on a Horizontal Surface

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Introduction

The use of daily solar radiation in climate and crop growth models and for planning active and passive solar energy systems requires an accurate assessment of the spatial distribution of daily solar radiation. Incident solar radiation is measured with pyranometers at relatively few stations in the U.S. Sunshine duration is measured at most 24-hour National Weather Service (NWS) stations, which has led to many studies on the use of percent possible sunshine to predict the incident solar radiation. Baker and Haines (1969) reviewed this literature and computed statistical regressions of solar radiation on percent possible sunshine for all NWS stations in the North Central U. S. for which both solar radiation and sunshine duration were measured. The equations were of the form

$$SR = R^*(a + bS) \qquad [1]$$

where SR is the estimated daily solar radiation at the earth's surface, R* is the extraterrestrial radiation, S is the percentage of possible sunshine, and a and b are fitted regression coefficients determined for each climatological week for the period of record, generally 1952-1966. For Indianapolis, their regression equations were generally associated with about 0.7 to 0.9 of the variance in daily solar radiation within weekly periods for which the regression coefficients were estimated.

Over the past few years models based on standard meteorological data have been proposed to simulate radiative transfer through clear and cloudy atmospheres (Atwater and Ball 1981; Davies, Schertzer and Nunez 1975; Suckling and Hay, 1977). Clear sky radiation estimates are generally within 5% of the measured values. Cloud effects are included by using a transmittance function presented by Manabe and Strickler (1964). To use this function the reported cloud type and amount are required. Considerable work is underway attempting to estimate daily solar radiation at the surface from satellite data. Encouraging results were obtained by Gautier, Diak and Masse (1980) and by Brakke and Kanemasu (1981) for both clear sky and cloudy conditions.

The objective of this study is to examine and develop methods for estimating daily totals of SR in near real time over Indiana, utilizing available information for percent possible sunshine, total opaque cloudiness, and observations of cloud heights and amounts reported hourly for aviation purposes.

Data and Procedures

There are two fairly long records of SR observations in Indiana. The NWS station at Indianapolis International Airport began in 1952 and has continued, with sizeable breaks in the records, to the present. Sunshine duration measurements and cloud observations are also taken at the same location. An Eppley pyranometer was installed at the cooperative NWS-Purdue University climatological station at the Agronomy Farm, 6 miles NW of West Lafayette, in 1957. This record of daily SR has large gaps until 1968, after which hourly solar radiation and also duration of sunshine have been measured continuously. There were no cloud observations at the Agronomy Farm, but these are taken at the

Federal Aviation Administration (FAA) station at the Purdue University Airport, about 5 miles SE of the Agronomy Farm. Sunshine duration is measured only at two other locations in Indiana, the NWS airport stations at Fort Wayne and Evansville. Cloud observations are also taken at these stations and also at Muncie, South Bend, Terre Haute, Bloomington, and Grissom AFB. At each of these stations, observations of the tenths of total and total opaque sky cover are taken, as well as the tenths of clouds at low, middle and high levels.

Three methods for estimating SR were examined and compared for their relative accuracy: (1) using percent of possible sunshine, (2) using the total opaque cloudiness, and (3) using the cloud amounts reported at each level with estimated transmission coefficients. For (1) the regression model in Eq. 1, with measurements of S and the coefficients estimated by Baker and Haines (1969), was used to estimate the SR for independent periods of record. The year 1968 was selected because solar radiation measurements were available at both Indianapolis and West Lafayette 6 NW. For (2), preliminary analysis showed that the total opaque sky cover was better correlated with SR than was the total sky cover. A day with high thin cirrostratus overcast may cause only slight diminution of the solar radiation but a heavy overcast will greatly reduce SR. Only total cloudiness is published in local Climatological Data for NWS staitons, and neither total nor total opaque cloudiness is sent in the hourly aviation reports. One has to use manuscript hourly observational forms to obtain hourly opaque cloudiness (OC) data. Since 10 tenths (overcast) OC will cause greater SR reduction during noon hours than during morning or evening hours, a daily weighted average opaque cloudiness ($\overline{OC}$) was computed by weighting the OC for each hour by the ratio of the SR for the respective hour (i) to the total daily SR received on a clear day,

$$\overline{OC} = \sum_{i=5AM}^{9PM} (SR_i/SR_{day})OC_i \qquad [2]$$

Scatter diagrams of SR on $\overline{OC}$ showed a curvilinear pattern (Figure 2) and quadratic regressions were fitted. The regressions of SR on $\overline{OC}$, as well as the SR_i/SR_{day} weights used in [2], were computed for calendar periods with similar solar declination.

For method (3) the equation used to compute irradiance (I) at the surface is

$$I = I_0 \cos\theta T_R T_g T_w T_a T_c (1 + r_s r_c) \qquad [3]$$

where I_0 is the solar constant, θ the zenith angle, T the transmittance after Rayleigh scattering (R), absorption by permanent gases (g) and water vapor (w), absorption and scattering by aerosols (a), and absorption and reflectance from clouds (c). The albedo coefficients in [3], r_s and r_c, correspond to a single reflectance from the earth's surface and cloud cover, respectively. The rate at which solar radiation is received outside the earth's atmosphere on a surface normal to the incident radiation (extraterrestial radiation) varies slightly throughout the year and is given by

$$I_0 = 1353 \text{ W/m}^2 [1 + 0.034 \cos [2\pi(n - 1)/365)]$$

where n is the Julian day.

An empirical relationship to account for the effects of Rayleigh scattering and absorption by permanent gases was given by Kondratyev (1969). Atwater and Brown (1974) later modified this expression to account for the isotropic nature of

Rayleigh scattering with one-half of the scattered radiation being in the forward direction. This formula is given by

$$T_R T_g = 1.021 - 0.084\,[m\,(949\,p \times 10^{-5} + 0.051)]^{1/2}$$

where p is the surface pressure in kPa, and m is the air mass thickness coefficient, given by

$$m = 35/(1224 \cos^2 \theta + 1)^{1/2}$$

A formula by McDonald (1960) is used to account for water vapor absorption. The expression is

$$T_w = 1 - 0.077(um)^{0.3}$$

where u is the precipitable water and m again is the air mass. An empirical relationship by Smith (1966), used to estimate the precipitable water (u) from the surface dew point temperature (T_d), is $u = \exp[(0.1133 - \ln(\lambda + 1)) + 0.0393\ T_d]$, where λ is an empirically-derived constant for different seasons and latitudes. Since standard meteorological observations cannot be used to estimate the aerosol attenuation, T_a was evaluated as a residual from clear sky conditions. The relation follows an expression given by Houghton (1954) which is

$$T_a = 0.95^m.$$

The transmittance function used by Manabe and Strickler (1964) for multiple cloud layers is

$$T_c = \prod_{j=1}^{n} [1 - (1 - t_j)C_j], \qquad [4]$$

where n is the number of cloud layers, t_j the transmission for the jth cloud layer, and C_j the coverage of the jth layer.

The cloud transmission coefficients were estimated empirically by substituting [4] in [3] and solving for t_j for single cloud layers at various heights and coverages. Hourly solar radiation values taken at the Purdue Agronomy Farm were used in conjunction with the corresponding hourly cloud observations from the Purdue Airport FAA station, approximately 5 miles SE of the Agronomy Farm. Populations of transmission coefficients were generated for various cloud heights and coverages. The frequency distributions were fitted with a beta distribution (Yao, 1969), but only means of each distribution were used in [4] to calculate the hourly incident solar radiation. The hourly values were then summed to obtain daily SR estimates.

Results and Discussion

The daily SR values predicted with S and regression coefficients from Baker and Haines (1969) in [1], have been plotted against the respective measured SR in Figure 1 for Indianapolis. This and other tests for Lafayette and Indianapolis yielded coefficients of determination (r^2) near or above 0.9 with regression slopes near 1.

A plot of the measured SR on the daily $\overline{OC}$, computed with [2], is shown in Figure 2 for the period 15 May to 24 July, 1968, roughly a month before to a month after the summer solstice. The fitted quadratic regression was associated with 0.9 of the variance in the measured SR. The fitted regressions for other intervals of the growing season with similar declination are shown in Figure 3. Almost all of these regressions were associated with 0.9 or more of the variance in daily SR.

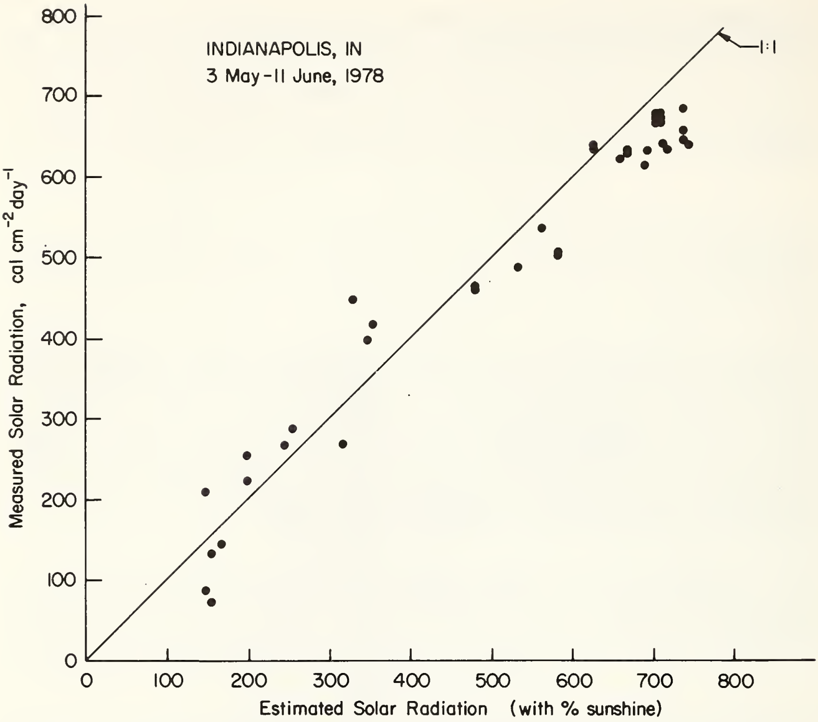

FIGURE 1. *Scatter diagram of measured daily solar radiation on that predicted with percentage possible sunshine and the regression equation from Baker and Haines (1969) for Indianapolis, IN, 3 May—11 June, 1978.*

Since the third method is more deterministic and includes more variables, some additional description of the procedural results is included. For example, a histogram of the transmission coefficients, t_i, calculated for cirrus layers (cloud heights greater than 18,000 ft.) is shown in Figure 4. The negatively-skewed distribution has a mean t_i of 0.67. Histograms were also plotted for the other cloud layers, and each was fitted with the Beta distribution (Yao, 1969). The means of the empirical distributions for the corresponding cloud layers used in [4] are shown in Table 1. Note that thin cirrus has a mean transmission of 0.81. An independent test

TABLE 1. *Means of the cloud transmission coefficient ($\bar{t}$) for the different cloud layer heights and overcast (ovc), broken (bkn), or scattered (sct) sky conditions.*

Cloud Layer		
Height (thousands ft.)	Coverage	t
0- 6	(ovc)	.30
0- 6	(bkn, sct)	.68
6-12	(ovc)	.43
6-12	(bkn, sct)	.67
12-18	(bkn, sct, ovc)	.44
>18	(bkn, sct, ovc)	.67
"thin" >18	(bkn, sct, ovc)	.81

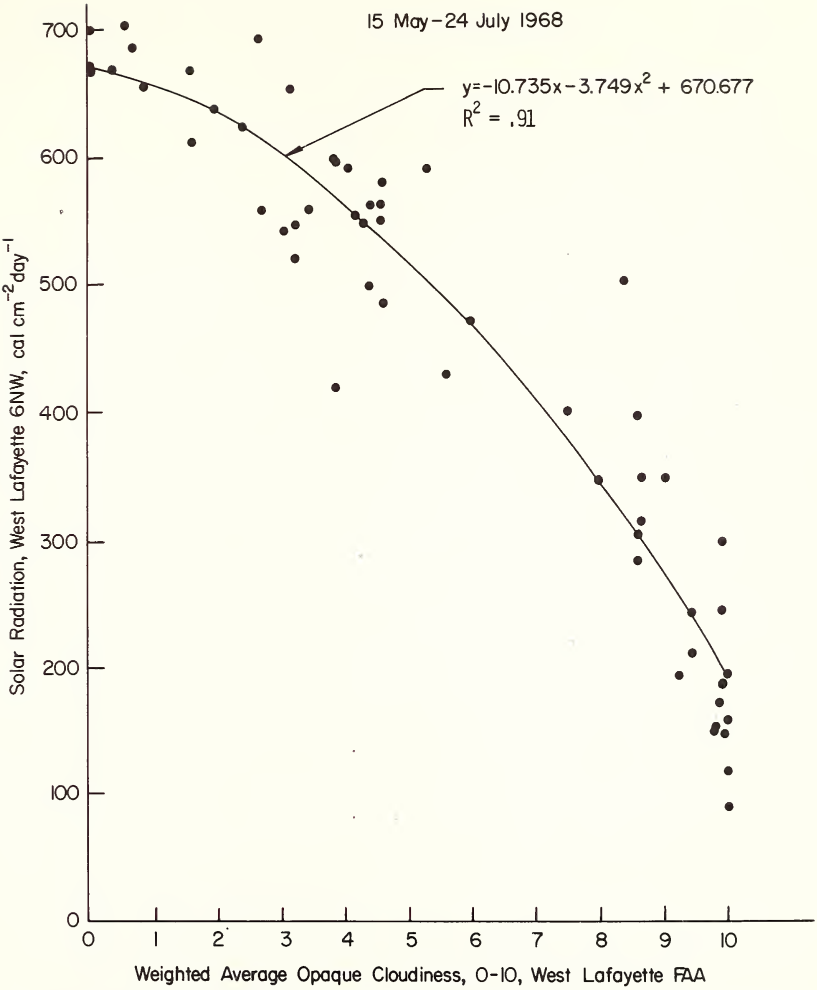

FIGURE 2. *Scatter diagram of measured daily solar radiation on weighted opaque cloudiness, West Lafayette, IN, 15 May—24 July 1968, with fitted quadratic regression.*

on 60 days randomly selected from Indianapolis in 1980 showed excellent agreement between the predicted and observed solar radiation, as shown in Figure 5. Similar results were obtained for a 120-day sample for West Lafayette, 1976 (Figure 6). Both high and low values of daily solar radiation were estimated well with a root mean square error of 1.88 MJ m^{-2} day^{-1} (45 cal cm^{-2} day^{-1}) for Indianapolis and 1.38 (33) for West Lafayette.

All three methods were tested and compared for a sample of 60 days for West Lafayette, 1976. Five days were selected from each month to give a representative sample. The mean absolute error, mean error, and root mean square error were computed for each method and are given in Table 2. The largest mean absolute

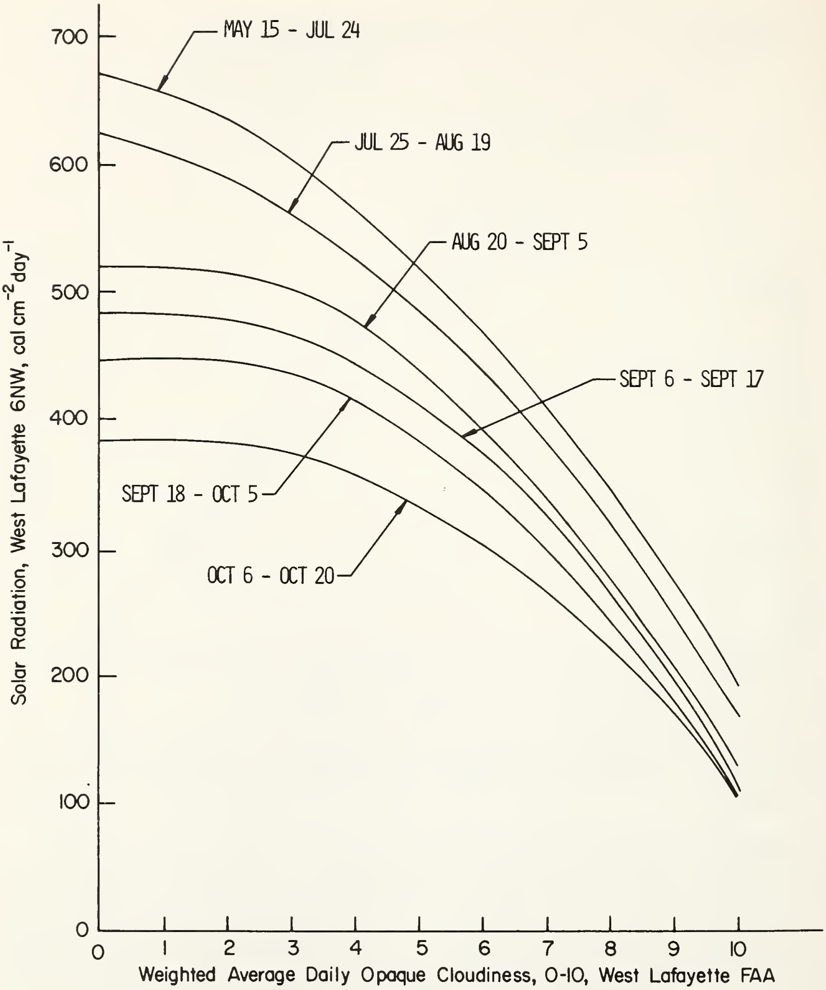

Figure 3. *Fitted regression equations (as in Fig. 2) for indicated period of similar solar declination, West Lafayette, IN, 1968.*

error was 2.19 MJ m^{-2} (52.4 cal cm^{-2}) per day for the first method (using percent possible sunshine). The weighted average opaque cloudiness yielded a mean absolute error of 1.5 MJ m^{-2} day^{-1}. The third method did the best, giving a mean absolute error of 1.06 MJ m^{-2} day^{-1} (25.4 cal cm^{-2} day^{-1}). The root mean square error with method 1 was the largest (2.64 MJ m^{-2} day^{-1}) and with method 3 the lowest (1.4 MJ m^{-2} day^{-1}). The mean errors for all methods approached zero, indicating all are unbiased. SR in Indiana can be estimated generally within 2.0 MJ m^{-2} day^{-1}.

Summary

All three methods presented were shown to be effective in estimating daily solar radiation for agriculture and energy-related purposes. Predictions with all

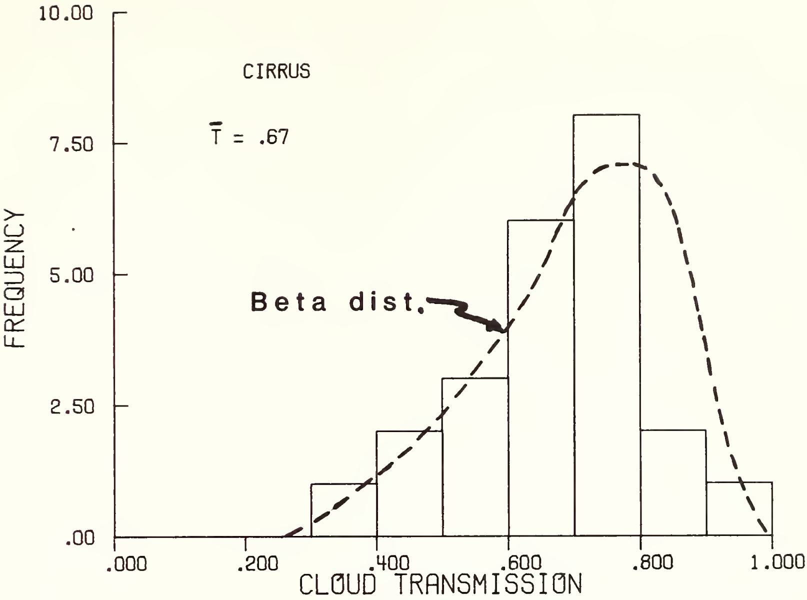

FIGURE 4. *Histogram of solar radiation transmission coefficients for cirrus clouds, West Lafayette, IN 1970-1975.*

three methods generally were with 2.0 MJ m^{-1} day^{-1} of the measured daily solar radiation. since it is unlikely that the present radiation network will be increased, cloud observations can be utilized to estimate daily solar radiation, at several additional locations in Indiana for which SR data are not available.

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TABLE 2. *Mean error* $\left(\bar{e} = \left|\sum_{i=1}^{n} \textit{predicted SR - observed SR}\right| /n\right)$, *mean absolute error* ($|\bar{e}|$), *and root mean square error* ($\sqrt{e^2/n}$) *for three methods of estimating solar radiation using (1) daily percent possible sunshine, (2) weighted daily opaque cloudiness, and (3) hourly reports of cloud heights and amounts for five days in each month, January-December, 1976, West Lafayette, IN.*

Method	Mean error	Mean absolute error MJ m^{-2} day^{-1} (cal cm^{-2} day^{-1})	Root mean square error
1	-0.38(-9.04)	2.19(52.4)	2.64(63.2)
2	-0.18(-4.20)	1.45(34.6)	1.98(47.5)
3	-0.16(-3.80)	1.06(25.4)	1.38(33.1)

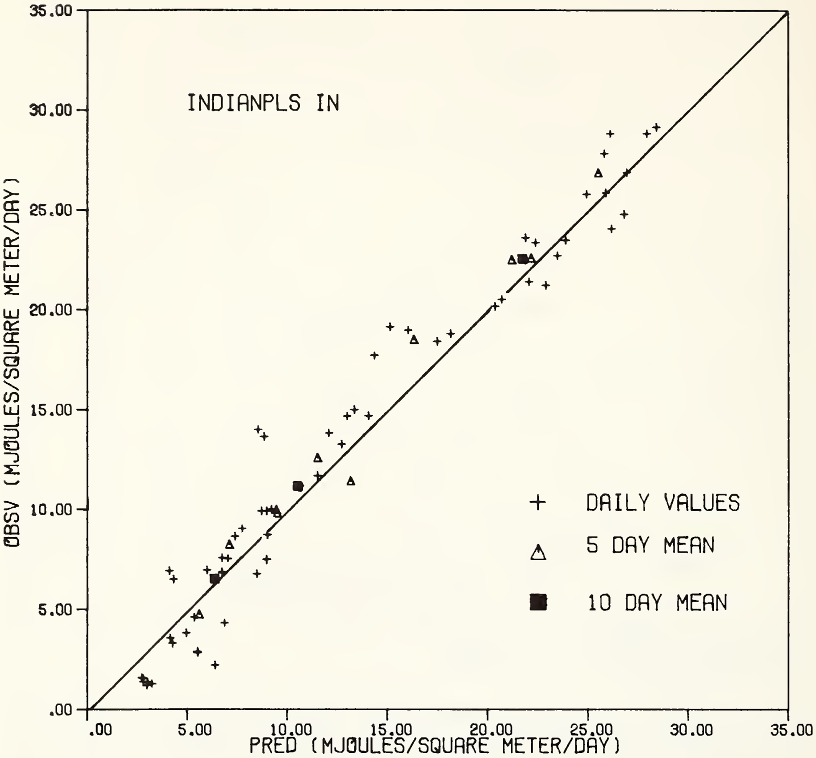

FIGURE 5. *Scatter diagram of measured daily solar radiation on that predicted with hourly cloud amounts and transmission coefficients for an independent random sample, Indianapolis, IN, 1980.*

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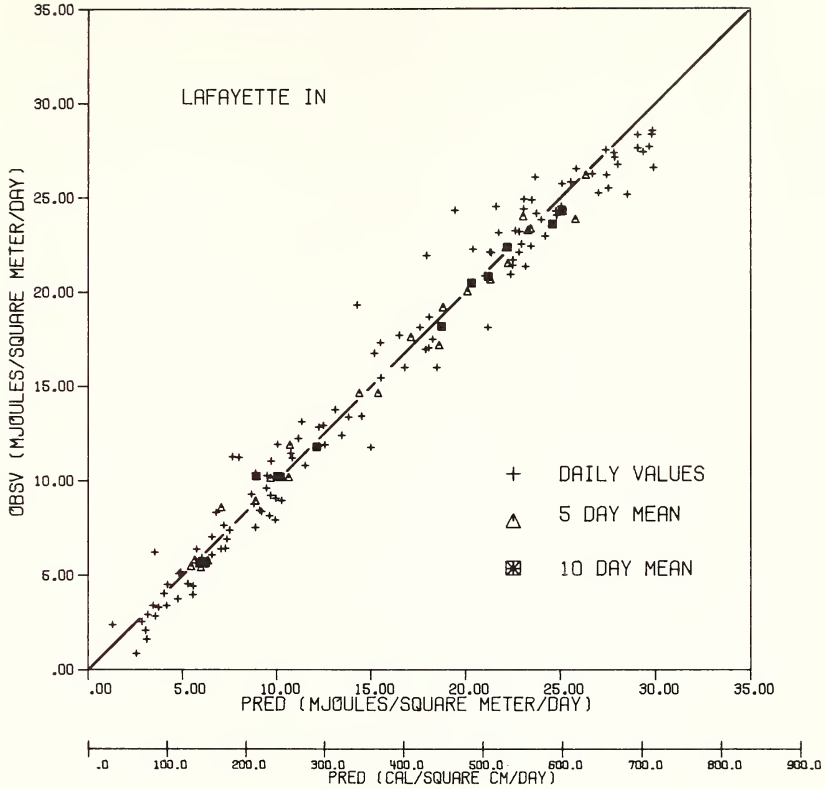

FIGURE 6. *Scatter diagram of measured daily solar radiation on that predicted with hourly cloud amount and transmission coefficients for an independent random sample, West Lafayette, IN, 1976.*

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