Feasibility of Midwest Crustal Studies Based on Earthquake Surface Wave Ellipticities

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Abstract

Because of the scarcity of seismograph stations in Indiana and surrounding states, there is a need for the development of a method for determining crustal structures from a single isolated station. A method utilizing Rayleigh wave ellipticity, as developed by Boore and Toksoz, was applied to data from the Indiana University seismograph station. Ellipticity is based on the spectral ratios of the Rayleigh wave particle motion, and the calculations are made using standard Fourier techniques. Preliminary comparisons with computer models of the Midwest suggest the feasibility of this method as a useful tool in studying the local crustal configuration.

Introduction

The development of a technique for determination of crustal structure from earthquake seismograms obtained at a single seismograph station is desirable, especially for areas where seismograph stations are scarce. One such technique utilizing ellipticity (ratio of radial to vertical particle displacement) of Rayleigh waves was proposed by Boore and Toksoz (2), but has since received very little attention. The purpose of this paper is to briefly review the ellipticity technique, to present the results of its first application in the U. S. Midwest and to indicate the direction of further research.

Seismic surface waves and body waves conventionally provide two independent sources of information (1) on the structure of the earth's crust and upper mantle (Fig. 1). Rayleigh wave velocities and Rayleigh wave ellipticity also provide two independent sources of information related to the structure of the earth (2). Study of the ellipticity method is justified because there has been only one published attempt to apply the method to observational data (2), and because resulting models represent a local average of crustal properties.

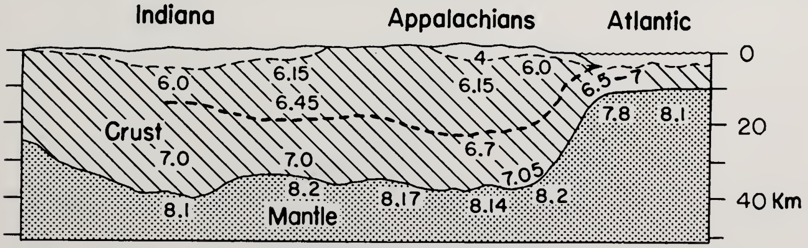

FIGURE 1. *Cross section of Eastern and Midwestern U. S. crust showing the Moho (solid line) and Conrad (dashed line) discontinuities. Modified from Wyllie (4).*

Although the Boore-Toksoz study theoretically demonstrated that ellipticity provides information independent of phase velocity, their observational ellipticity data (Fig. 2) showed too much scatter to enable them to derive a crustal model. Instead, they first used phase velocity data to derive several models and then used the ellipticity data as a constraint to choose the final model. Scatter in the observational data may have been caused by the complex geology in the area studied (LASA seismic array of Eastern Montana). Application of the method in a less complex geologic setting may be more successful and further establish the feasibility of the ellipticity method. A brief and elementary review of the theory follows.

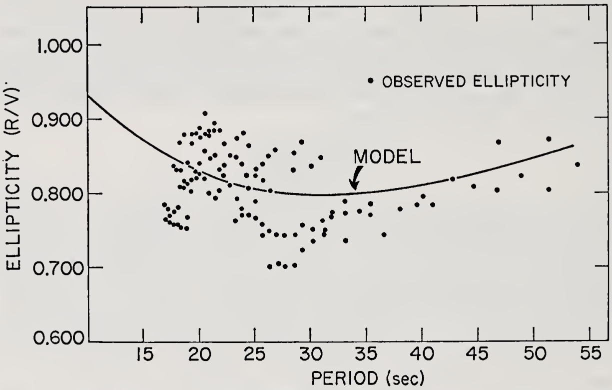

Figure 2. *Ellipticity data from Boore-Toksoz study in Eastern Montana. Dots represent observed ellipticity. Solid line represents theoretical ellipticity calculated from a model. After Boore and Toksoz (2).*

The Ellipticity Technique

The theory for Rayleigh waves on a semi-infinite elastic solid was given by Lord Rayleigh in 1885. He demonstrated the existence of waves whose amplitudes decrease exponentially with depth. Particle motion is restricted to the vertical plane and is retrograde elliptical with respect to the direction of propagation (Fig. 3). For the homogeneous and isotropic half-space, the radial particle displacement (horizontal particle

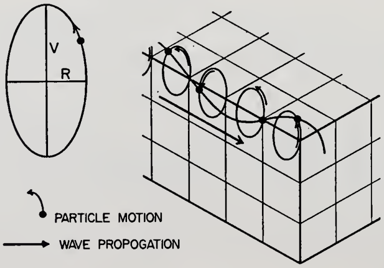

Figure 3. *Diagrammatic representation of particle motion of a Rayleigh wave with close up of elliptical orbit. V represents vertical particle motion. R represents radial particle motion.*

displacement toward or away from wave propagation direction) is about 0.666 times the vertical particle displacement ($R/V = 0.666$). This ratio is the definition of ellipticity. Radial motion vanishes at 0.192 of a wavelength and below this depth the motion becomes elliptical prograde. The velocity of Rayleigh waves for the homogeneous isotropic half-space is approximately 9/10 of the velocity of shear waves in the medium.

When a layered earth is introduced, Rayleigh waves exhibit disperson, that is, the phase and group velocities become functions of period (or frequency). Thus, particle motion, and hence ellipticity (ratio of particle displacements) also become a function of period as defined in Equation 1.

$$E(T) = \frac{|R(T)|}{|V(T)|} \tag{1}$$

T = period of Rayleigh waves

E = ellipticity

R = radial Rayleigh wave particle displacement

V = vertical Rayleigh wave particle displacement

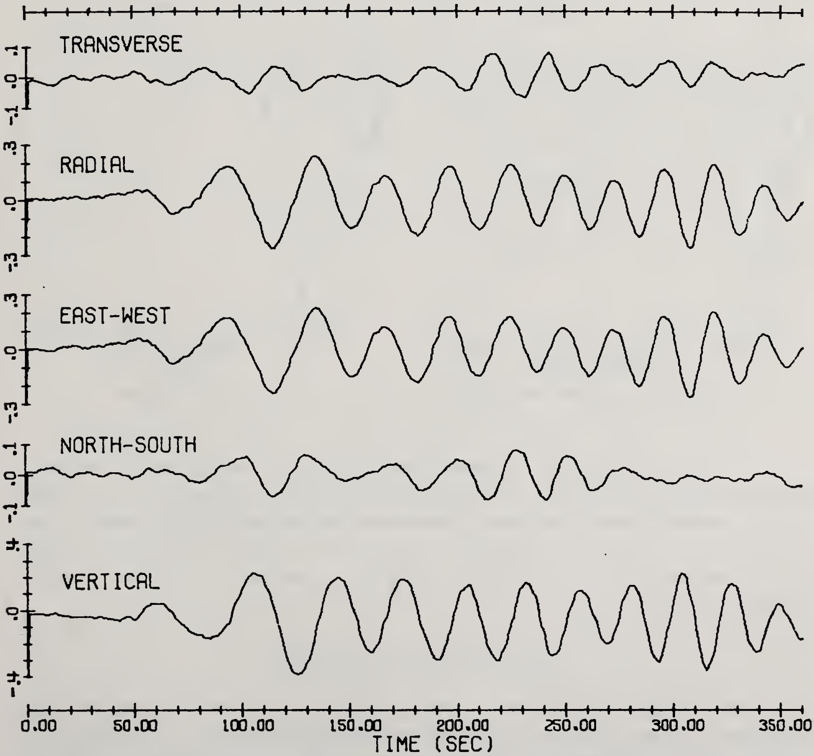

FIGURE 4. *Digitized portions of the original (NS, EW, Vertical) seismograms and calculated radial and transverse seismograms.*

Ellipticity is defined in terms of vertical and radial motion, therefore, it is necessary to combine the north-south and the east-west seismograms to produce the radial seismogram. This may be accomplished by computer implementation of a simple two dimensional coordinate transformation. Digitized portions of the original records along with the calculated radial and transverse seismograms are shown in Figure 4. A transverse seismogram represents motion perpendicular to radial motion.

Because the radial Rayleigh wave train R(t) and the vertical Rayleigh wave train V(t) are recorded as time series, it is necessary to transform the recorded wave motion to displacement as a function of frequency or period. Thus the ellipticity (Equation 1) is the ratio of two amplitude spectra (Fig. 5). In practice the transformation was accomplished by Fourier analysis performed on a CDC 6600 digital computer and using a Fast Fourier Transform algorithm.

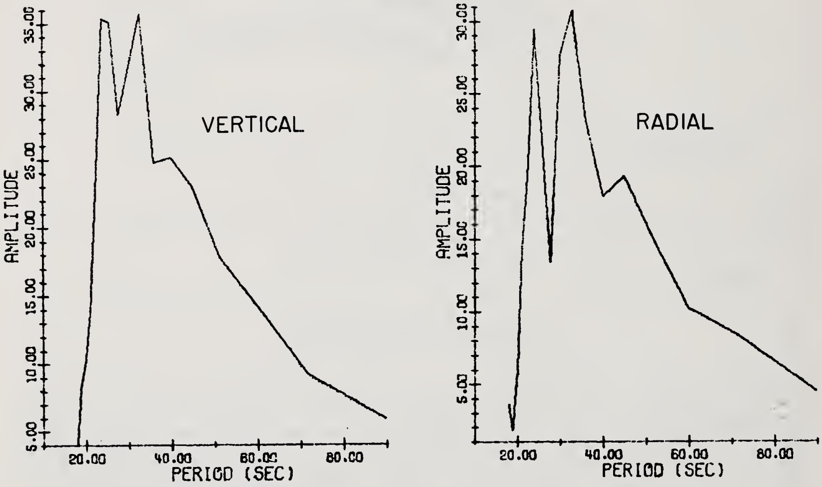

FIGURE 5. *Amplitude spectra of the vertical and radial seismograms.*

It was previously noted that layering introduced frequency dependence of ellipticity. Further, the ellipticity values at a particular period depend upon the type of layering introduced in the earth model, including the elastic parameters within each layer. These properties allow comparison of theoretical ellipticity (model) curves to observed ellipticity curves.

A Preliminary Application

The ellipticity technique was applied to a set of seismograms recorded at the Indiana University seismograph station at Bloomington, Indiana. The Earthquake studied occurred on February 12, 1972, and was located at 15.3 S. Latitude and 173.4 W. Longitude in the Tonga

Islands. Its origin time was 18 hours, 51 min, and 57 sec and it had a magnitude of 5.9 on the Richter Scale.

A comparison of observed and model ellipticity is given in Figure 6. Observed ellipticity was obtained (using Equation 1) from smoothed spectra of the radial and vertical seismograms. Smoothing was performed by a five point equal-weight moving average operator. Theoretical ellipticity was calculated for two crustal models of Midwestern United States. Model 1 was derived from surface wave phase velocity (3). Important parameters include density (ρ), P wave velocity (α), and S wave velocity (β) for each layer of given thickness.

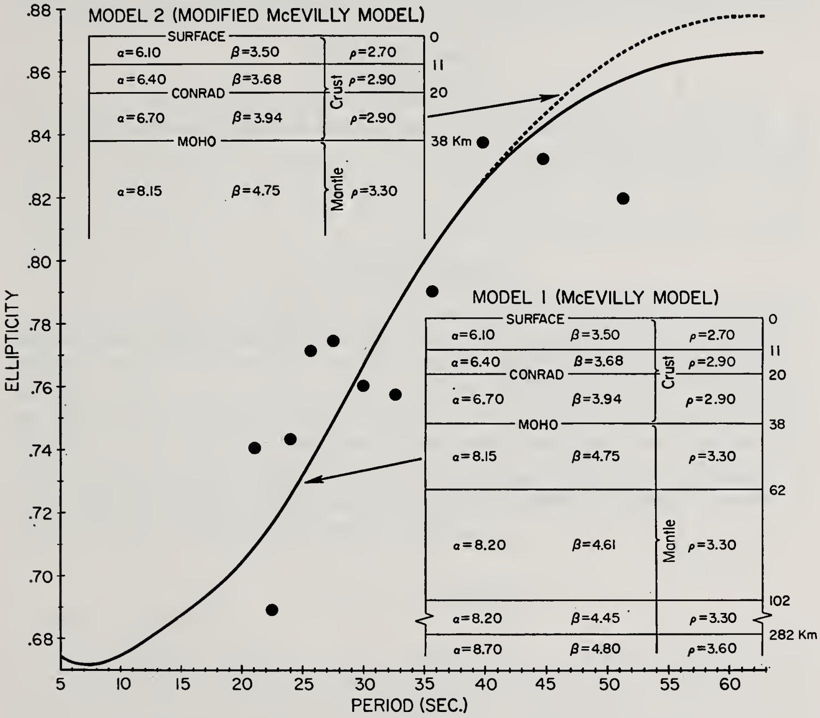

FIGURE 6. *Comparison of observed and model ellipticity. Model 1 represents the crustal model of McEvilly (3). Model 2 represents a modified version of Model 1.*

Model 2 was derived from Model 1 by retaining only crustal layering and using constant values within the mantle. The second model is introduced to show that in the period range studied, the layering within the mantle has only a slight effect on the theoretical curve. This observation suggests that one need only consider changes in the crustal parameters in model calculations. Other models are being examined to determine the effect of changes in elastic parameters of the various layers.

Conclusions

This study represents a preliminary investigation. However, general agreement between the theoretical and observed ellipticity (Fig. 6) is the first data to support this technique as a feasible method for determination of crustal structure in the Midwest.

Further studies using this method have been planned and are the object of current research. Future research will include examination of additional earthquake data and additional models for the present crustal study, examination of shorter period data for earthquakes and propagation paths within the Illinois Basin, and the effect of direction of approach (azimuth) of the surface waves. Filtering techniques for further refinement of the data are now being considered.

Acknowledgments

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